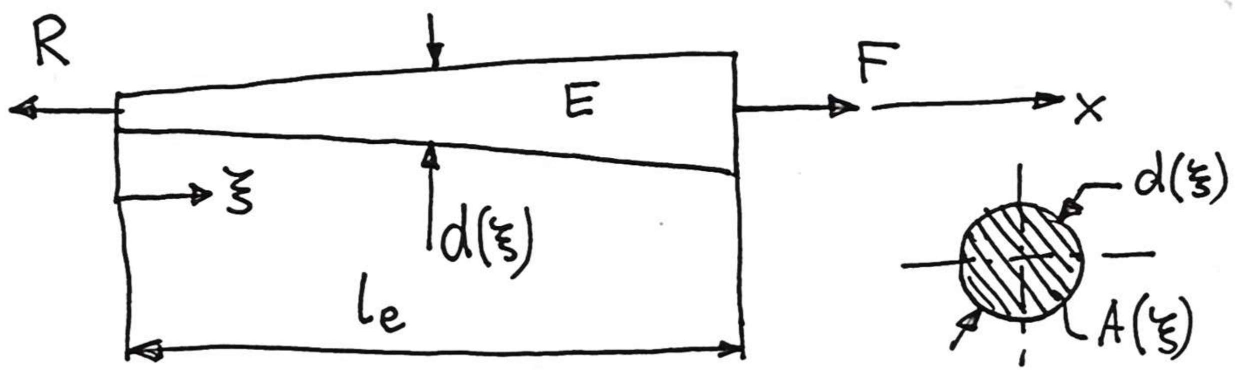
Warsaw University of Technology
Faculty of Power and Aeronautical Engineering
WARSAW UNIVERSITY OF TECHNOLOGY
Institute of Aeronautics and Applied Mechanics

Finite element method (FEM)

Conical bar element

05.2021

CONICAL BAR ELEMENT WITH A CIRCULAR CROSS-SECTION



DIAMETER $d(\xi) = \frac{d_3 - d_1}{l_e} \cdot \xi + d_1$ (LINEAR FUNCTION OF COORD. ξ)

$d_1 = d(0) \quad , \quad d_3 = d(l_e)$

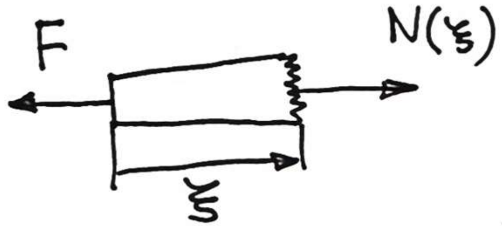
CROSS-SECTIONAL AREA : $A(\xi) = \frac{\pi \, d(\xi)^2}{4} = a\xi^2 + b\xi + c$ where:

$a = \frac{\pi (d_3 - d_1)^2}{4 l_e^2} \quad , \quad b = \frac{\pi (d_3 - d_1) d_1}{2 l_e} \quad , \quad c = \frac{\pi d_1^2}{4}$

ANALYTICAL SOLUTION

$$\Sigma F_x = 0 : \quad -R + F = 0$$

$$R = F$$



INTERNAL FORCE:

$$N(\xi) - F = 0 \Rightarrow N(\xi) = F = \text{const.}$$

STRESS : $$\sigma_x(\xi) = \frac{N(\xi)}{A(\xi)} = \frac{F}{A(\xi)}$$

STRAIN : $$\varepsilon_x(\xi) = \frac{\sigma_x(\xi)}{E} = \frac{F}{E \cdot A(\xi)}$$

DISPLACEMENT :

$$u(\xi) = \int_0^{\xi} \frac{F}{E\,A(x)}\,dx = \frac{F}{E}\int_0^{\xi} \frac{dx}{ax^2 + bx + c}$$

$$\Delta = b^2 - 4ac = \frac{\pi^2 (d_3-d_1)^2 d_1^2}{4le^2} - 4 \cdot \frac{\pi (d_3-d_1)^2}{4le^2} \cdot \frac{\pi d_1^2}{4} = 0$$

$$x_0 = -\frac{b}{2a} = -\frac{\pi (d_3-d_1) d_1 \cdot 4le^2}{2le \cdot 2 \cdot \pi (d_3-d_1)^2} = -\frac{le \cdot d_1}{d_3-d_1}$$

$$\Rightarrow ax^2 + bx + c = a(x-x_0)^2$$

$$u(\xi) = \frac{F}{E} \int_0^{\xi} \frac{dx}{a(x-x_0)^2} = \left| \begin{array}{l} x - x_0 = y \\ dx = dy \\ x = 0 \Rightarrow y = -x_0 \\ x = \xi \Rightarrow y = \xi - x_0 \end{array} \right| = \frac{F}{Ea} \int_{-x_0}^{\xi - x_0} \frac{dy}{y^2} =$$

$$= \frac{F}{Ea} \left(\frac{y^{-2+1}}{-2+1} \right) \Bigg|_{-x_0}^{\xi - x_0} = -\frac{F}{Ea} \cdot \left(\frac{1}{y} \right) \Bigg|_{-x_0}^{\xi - x_0} =$$

$$= -\frac{F}{Ea} \left(\frac{1}{\xi - x_0} + \frac{1}{x_0} \right) = -\frac{F}{Ea} \left(\frac{\xi - x_0 + x_0}{(\xi - x_0) x_0} \right) =$$

$$= u(\xi) = -\frac{F\xi}{Ea}\cdot\frac{1}{\left(\xi+\frac{le\cdot d_1}{d_3-d_1}\right)\cdot\left(-\frac{le\,d_1}{(d_3-d_1)}\right)} = \frac{F\xi}{Ea}\cdot\frac{1}{\frac{(\xi\cdot(d_3-d_1)+le\,d_1)\,le\,d_1}{(d_3-d_1)^2}} =$$

$$= \frac{F\xi\,(d_3-d_1)^2\cdot 4\,le^2}{E\cdot\pi\,(d_3-d_1)^2\cdot(\xi(d_3-d_1)+le\,d_1)\,le\,d_1} = \frac{4\,F\,le\,\xi}{\pi\,E\,d_1\,(\xi\,(d_3-d_1)+le\,d_1)}$$

$$u(0) = 0\ ,$$

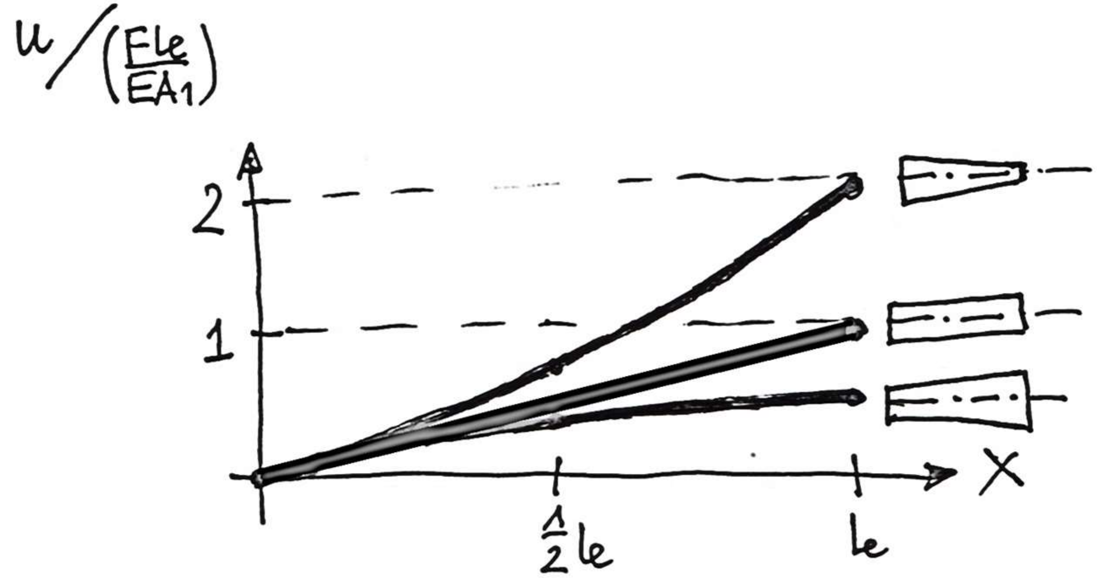
$$u\left(\frac{le}{2}\right) = \frac{4\,F\,le\,\frac{le}{2}}{\pi E d_1\left(\frac{le}{2}(d_3-d_1)+le\,d_1\right)} = \frac{2\,F\,le^2}{\pi E d_1\,le\left(\frac{d_3}{2}+\frac{d_1}{2}\right)} = \frac{4\,F\,le}{\pi\,E\,d_1\,(d_1+d_3)}\ ,$$

$$u(le) = \frac{4\,F\,le^2}{\pi\,E\,d_1\left(le(d_3-d_1)+le\,d_1\right)} = \frac{4\,F\,le}{\pi E\,d_1\,d_3}$$

$\frac{d_3}{d_1}$	$u\left(\frac{l_e}{2}\right) / \frac{F l_e}{E A_1}$	$u(l_e) / \frac{F l_e}{E A_1}$
$\frac{1}{2}$	$\frac{2}{3}$	2
1	$\frac{1}{2}$	1
2	$\frac{1}{3}$	$\frac{1}{2}$

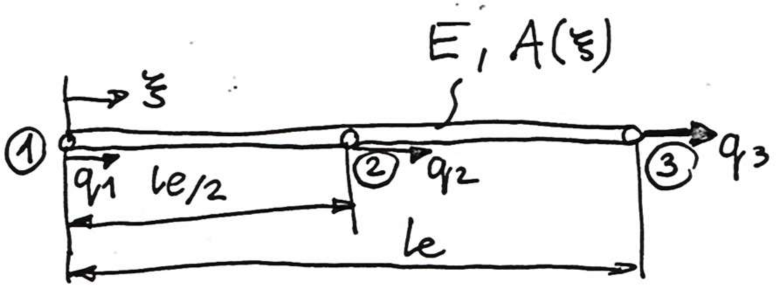
$$u(\xi) = \frac{4 F l_e \xi}{\pi E d_1 \left(\xi (d_3 - d_1) + l_e d_1 \right)}$$

$$A_1 = \frac{\pi d_1^2}{4}$$



$\Rightarrow$ $u(\xi)$ APPROXIMATION BY PARABOLA

FINITE ELEMENT



$$\{q\}_e = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}_e$$

ASSUMPTION : MIDSIDE NODE 2 at $l_e/2$.

$$u(\xi) = \alpha_1 + \alpha_2 \cdot \xi + \alpha_3 \cdot \xi^2 = 1 \cdot \alpha_1 + \xi \cdot \alpha_2 + \xi^2 \cdot \alpha_3 = \lfloor 1, \xi, \xi^2 \rfloor \cdot \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix}$$

$$u(0) = q_1 \implies q_1 = 1 \cdot \alpha_1 + 0 \cdot \alpha_2 + 0 \cdot \alpha_3$$

$$u\left(\frac{l_e}{2}\right) = q_2 \implies q_2 = 1 \cdot \alpha_1 + \frac{l_e}{2} \cdot \alpha_2 + \frac{l_e^2}{4} \cdot \alpha_3$$

$$u(l_e) = q_3 \implies q_3 = 1 \cdot \alpha_1 + l_e \cdot \alpha_2 + l_e^2 \cdot \alpha_3$$

$$\underset{3\times1}{\{q\}_e} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \frac{l_e}{2} & \frac{l_e^2}{4} \\ 1 & l_e & l_e^2 \end{bmatrix} \cdot \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \underset{3\times3}{[A]} \cdot \underset{3\times1}{\{\alpha\}} \quad , \quad \underset{3\times1}{\{\alpha\}} = \underset{3\times3}{[A^{-1}]} \cdot \underset{3\times1}{\{q\}_e}$$

$$[A]_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \frac{le}{2} & \frac{le^2}{4} \\ 1 & le & le^2 \end{bmatrix} \quad , \quad \det[A] = \frac{le}{2}\cdot le^2 - \frac{le^2}{4}\cdot le = \frac{le^3}{4}$$

$$c_{11} = \frac{le}{2}\cdot le^2 - \frac{le^2}{4}\cdot le = \frac{le^3}{4} \quad , \quad c_{12} = -\left(1\cdot le^2 - \frac{le^2}{4}\cdot 1\right) = -\frac{3}{4}le^2$$

$$c_{13} = 1\cdot le - \frac{le}{2}\cdot 1 = \frac{le}{2} \quad , \quad c_{21} = -(0\cdot le^2 - 0\cdot le) = 0$$

$$c_{22} = 1\cdot le^2 - 0\cdot 1 = le^2 \quad , \quad c_{23} = -(1\cdot le - 0\cdot 1) = -le$$

$$c_{31} = 0\cdot\frac{le^2}{4} - 0\cdot\frac{le}{2} = 0 \quad , \quad c_{32} = -\left(1\cdot\frac{le^2}{4} - 0\cdot 1\right) = -\frac{le^2}{4}$$

$$c_{33} = 1\cdot\frac{le}{2} - 0\cdot 1 = \frac{le}{2} \, .$$

$$[A^c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} l_e^3 & -3l_e^2 & 2l_e \\ 0 & 4l_e^2 & -4l_e \\ 0 & -l_e^2 & 2l_e \end{bmatrix}$$

$$[A^c]^T = \frac{1}{4}\begin{bmatrix} l_e^3 & 0 & 0 \\ -3l_e^2 & 4l_e^2 & -l_e^2 \\ 2l_e & -4l_e & 2l_e \end{bmatrix}$$

$$[A^{-1}] = \frac{1}{\det[A]}\cdot[A^c]^T = \frac{4}{l_e^3}\cdot[A^c]^T = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{l_e} & \frac{4}{l_e} & -\frac{1}{l_e} \\ \frac{2}{l_e^2} & -\frac{4}{l_e^2} & \frac{2}{l_e^2} \end{bmatrix}$$

NODAL APPROXIMATION / SHAPE FUNCTIONS

$$u(\xi) = \lfloor 1, \xi, \xi^2 \rfloor \cdot \underset{3\times1}{\{\alpha\}} = \lfloor 1, \xi, \xi^2 \rfloor \cdot \underset{3\times3}{[A^{-1}]} \cdot \underset{3\times1}{\{q\}_e} =$$

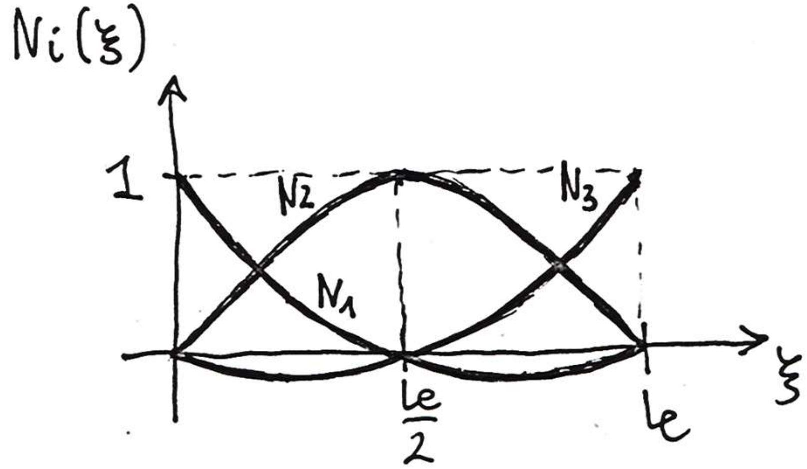
$$= \lfloor N_1(\xi), N_2(\xi), N_3(\xi) \rfloor \cdot \underset{3\times1}{\{q\}_e} = \underset{1\times3}{\lfloor N(\xi) \rfloor} \cdot \underset{3\times1}{\{q\}_e}$$

where :

$$N_1(\xi) = \frac{2\xi^2}{L_e^2} - \frac{3\xi}{L_e} + 1$$

$$N_2(\xi) = -\frac{4\xi^2}{L_e^2} + \frac{4\xi}{L_e}$$

$$N_3(\xi) = \frac{2\xi^2}{L_e^2} - \frac{\xi}{L_e}$$



$$\frac{dN_1}{d\xi} = \frac{4}{l_e}\left(\frac{\xi}{l_e} - \frac{3}{4}\right), \quad \frac{dN_2}{d\xi} = \frac{4}{l_e}\left(1 - \frac{2\xi}{l_e}\right), \quad \frac{dN_3}{d\xi} = \frac{4}{l_e}\left(\frac{\xi}{l_e} - \frac{1}{4}\right)$$

$$\varepsilon_x(\xi) = \frac{du}{d\xi} = \underset{1\times 3}{\left\lfloor \frac{dN}{d\xi} \right\rfloor} \cdot \underset{3\times 1}{\{q\}_e} \quad \text{or:} \quad \varepsilon_x(\xi) = \underset{1\times 3}{\lfloor q \rfloor_e} \cdot \underset{3\times 1}{\left\{\frac{dN}{d\xi}\right\}}$$

$$U_e = \frac{1}{2}\int_{\Omega_e} \underset{1\times 6}{\lfloor \varepsilon \rfloor} \cdot \underset{6\times 1}{\{\sigma\}} \, d\Omega_e = \frac{1}{2}\int_{\Omega_e} \varepsilon_x \cdot \sigma_x \, d\Omega_e = \frac{1}{2}\int_{\Omega_e} \varepsilon_x \cdot E \cdot \varepsilon_x \, d\Omega_e =$$

$$= \frac{1}{2}\int_{\Omega_e} \underset{1\times 3}{\lfloor q \rfloor_e} \cdot \underset{3\times 1}{\left\{\frac{dN}{d\xi}\right\}} \cdot E \cdot \underset{1\times 3}{\left\lfloor \frac{dN}{d\xi} \right\rfloor} \cdot \underset{3\times 1}{\{q\}_e} \, d\Omega_e = \left| \begin{array}{l} d\Omega_e = A(\xi) \cdot d\xi \\ A(\xi) = a\xi^2 + b\xi + c \end{array} \right| =$$

$$= \frac{1}{2} \underset{1\times 3}{\lfloor q \rfloor_e} \cdot E \cdot \int_0^{l_e} \underset{3\times 1}{\left\{\frac{dN}{d\xi}\right\}} \cdot \underset{1\times 3}{\left\lfloor \frac{dN}{d\xi} \right\rfloor} A(\xi) \, d\xi \cdot \underset{3\times 1}{\{q\}_e} = \frac{1}{2} \underset{1\times 3}{\lfloor q \rfloor_e} \cdot \underset{3\times 3}{[k]_e} \cdot \underset{3\times 1}{\{q\}_e}$$

STIFFNESS MATRIX

$$\underset{3\times 3}{[k]_e} = E \cdot \begin{bmatrix} \int_0^{l_e} N_1' N_1' A(\xi)\, d\xi & \int_0^{l_e} N_1' N_2' A(\xi)\, d\xi & \int_0^{l_e} N_1' N_3' A(\xi)\, d\xi \\ \int_0^{l_e} N_2' N_1' A(\xi)\, d\xi & \int_0^{l_e} N_2' N_2' A(\xi)\, d\xi & \int_0^{l_e} N_2' N_3' A(\xi)\, d\xi \\ \int_0^{l_e} N_3' N_1' A(\xi)\, d\xi & \int_0^{l_e} N_3' N_2' A(\xi)\, d\xi & \int_0^{l_e} N_3' N_3' A(\xi)\, d\xi \end{bmatrix}$$

$$k_{11} = E\int_0^{l_e} \frac{dN_1}{d\xi}\cdot\frac{dN_1}{d\xi}\cdot A(\xi)\,d\xi = E\int_0^{l_e} \frac{4}{l_e}\cdot\frac{4}{l_e}\cdot\left(\frac{\xi}{l_e}-\frac{3}{4}\right)^2\cdot\left(a\xi^2+b\xi+c\right)d\xi =$$

$$= \frac{16E}{l_e^2}\cdot\int_0^{l_e}\left(\frac{\xi^2}{l_e^2}-2\cdot\frac{\xi}{l_e}\cdot\frac{3}{4}+\frac{9}{16}\right)^2\left(a\xi^2+b\xi+c\right)d\xi =$$

$$= \frac{16E}{l_e^2}\int_0^{l_e}\left(\frac{a}{l_e^2}\xi^4+\frac{b}{l_e^2}\xi^3+\frac{c}{l_e^2}\xi^2-\frac{3a}{2l_e}\xi^3-\frac{3b}{2l_e}\xi^2-\frac{3c}{2l_e}\cdot\xi+\right.$$

$$\left.+\ \frac{9a}{16}\xi^2+\frac{9b}{16}\xi+\frac{9c}{16}\right)d\xi =$$

$$= k_{11} = \frac{16E}{l_e^2}\int_0^{l_e}\left(\frac{a}{l_e^2}\xi^4 + \left(\frac{b}{l_e^2} - \frac{3a}{2l_e}\right)\xi^3 + \left(\frac{c}{l_e^2} - \frac{3b}{2l_e} + \frac{9a}{16}\right)\xi^2 + \left(\frac{9b}{16} - \frac{3c}{2l_e}\right)\xi + \frac{9c}{16}\right)d\xi =$$

$$= \frac{16E}{l_e^2}\left(\frac{a\xi^5}{5l_e^2} + \left(\frac{b}{4l_e^2} - \frac{3a}{8l_e}\right)\xi^4 + \left(\frac{c}{3l_e^2} - \frac{b}{2l_e} + \frac{3a}{16}\right)\xi^3 + \left(\frac{9b}{32} - \frac{3c}{4l_e}\right)\xi^2 + \frac{9c}{16}\xi\right)\Bigg|_0^{l_e} =$$

$$= \frac{16E}{l_e^2}\left(\frac{al_e^3}{5} + \frac{bl_e^2}{4} - \frac{3al_e^3}{8} + \frac{cl_e}{3} - \frac{bl_e^2}{2} + \frac{3al_e^3}{16} + \frac{9bl_e^2}{32} - \frac{3cl_e}{4} + \frac{9cl_e}{16}\right) =$$

$$= 16E\left(\left(\frac{l_e}{5} - \frac{3l_e}{8} + \frac{3l_e}{16}\right)a + \left(\frac{1}{4} - \frac{1}{2} + \frac{9}{32}\right)\cdot b + \left(\frac{1}{3l_e} - \frac{3}{4l_e} + \frac{9}{16l_e}\right)\cdot c\right) =$$

$$= 16E\left(\left(\frac{16-30+15}{80}\right)al_e + \left(\frac{8-16+9}{32}\right)b + \left(\frac{16-36+27}{48l_e}\right)c\right) =$$

$$= 16E\left(\frac{al_e}{80} + \frac{b}{32} + \frac{7c}{48l_e}\right) = E\left(\frac{al_e}{5} + \frac{b}{2} + \frac{7c}{3l_e}\right)$$

$$K_{12} = E\int_0^{le} \frac{dN_1}{d\xi}\cdot\frac{dN_2}{d\xi}\cdot A(\xi)\,d\xi = -E\left(\frac{2a\,le}{5}+\frac{2b}{3}+\frac{8c}{3\,le}\right) = K_{21}$$

$$K_{13} = E\cdot\int_0^{le} \frac{dN_1}{d\xi}\cdot\frac{dN_3}{d\xi}A(\xi)\,d\xi = E\left(\frac{a\,le}{5}+\frac{b}{6}+\frac{c}{3le}\right) = K_{31}$$

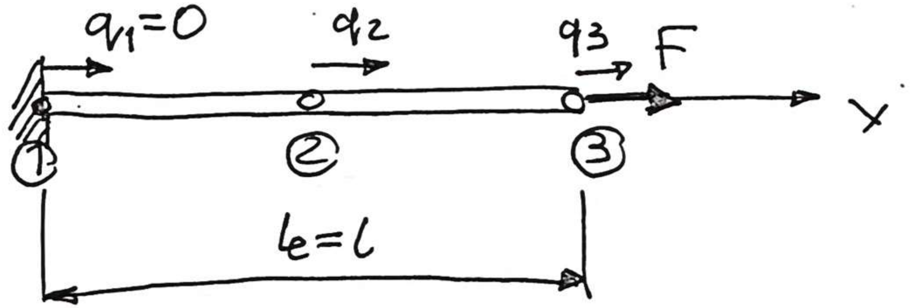
$$K_{22} = E\cdot\int_0^{le} \frac{dN_2}{d\xi}\cdot\frac{dN_2}{d\xi}A(\xi)\,d\xi = E\left(\frac{32a\,le}{15}+\frac{8b}{3}+\frac{16c}{3le}\right)$$

$$K_{23} = E\cdot\int_0^{le} \frac{dN_2}{d\xi}\cdot\frac{dN_3}{d\xi}A(\xi)\,d\xi = -E\left(\frac{26a\,le}{15}+2b+\frac{8c}{3le}\right) = K_{32}$$

$$K_{33} = E\cdot\int_0^{le} \frac{dN_3}{d\xi}\cdot\frac{dN_3}{d\xi}A(\xi)\,d\xi = E\left(\frac{23a\,le}{15}+\frac{11b}{6}+\frac{7c}{3le}\right)$$

$$\underset{3\times3}{[K]_e} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \quad ; \quad \det \underset{3\times3}{[K]_e} = 0$$

1°) 1 FE:



$$\underset{3\times3}{[K]} = \underset{3\times3}{[k]_e}, \quad \underset{3\times1}{\{q\}} = \underset{3\times1}{\{q\}_e}, \quad \underset{3\times1}{\{F\}} = \begin{Bmatrix} R_1 \\ 0 \\ F \end{Bmatrix}, \quad q_1 = 0$$

$$\begin{bmatrix} k_{22} & k_{23} \\ k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ F \end{Bmatrix}, \quad k_{32} = k_{23}$$

$$k_{22} q_2 + k_{23} q_3 = 0 \implies q_3 = -\frac{k_{22}}{k_{23}} q_2$$

$$k_{23} q_2 + k_{33} q_3 = F$$

$$k_{23} q_2 - \frac{k_{22}}{k_{23}} q_2 = F \implies \frac{k_{23}^2 - k_{22}}{k_{23}} q_2 = F$$

SOLUTION

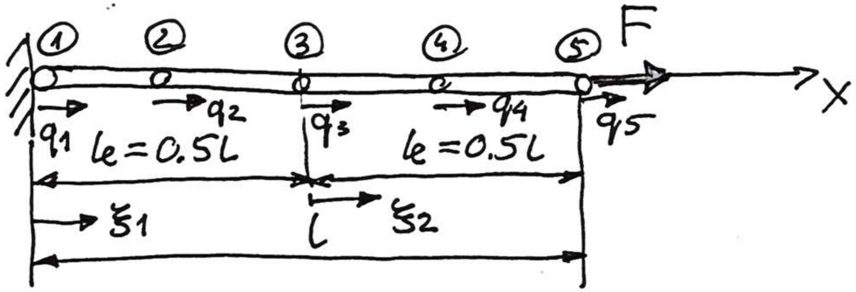
$$q_2 = \frac{F k_{23}}{k_{23}^2 - k_{22}} \quad , \quad q_3 = - \frac{F k_{22}}{k_{23}^2 - k_{22}}$$

$$u(\xi) = \underset{1\times 3}{\lfloor N(\xi) \rfloor} \cdot \underset{3\times 1}{\{q\}_1} = \lfloor N_1(\xi), N_2(\xi), N_3(\xi) \rfloor \cdot \begin{Bmatrix} 0 \\ q_2 \\ q_3 \end{Bmatrix}_1 = N_2(\xi) \cdot q_2 + N_3(\xi) \cdot q_3$$

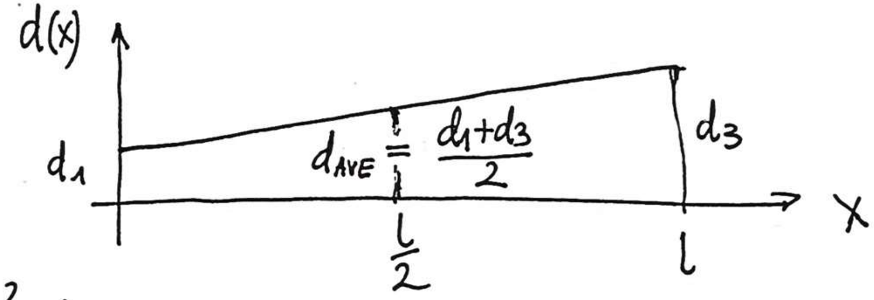
$$\varepsilon_x(\xi) = \frac{dN_2}{d\xi} \cdot q_2 + \frac{dN_3}{d\xi} \cdot q_3$$

$$\sigma_x(\xi) = E \cdot \varepsilon_x(\xi) \quad , \quad N(\xi) = \sigma_x(\xi) \cdot A(\xi)$$

2°) 2 FEs:



$$\{q\}_1 = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}, \quad \{q\}_2 = \begin{Bmatrix} q_3 \\ q_4 \\ q_5 \end{Bmatrix}, \quad \underset{5\times1}{\{q\}} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{Bmatrix}$$



$$A_1(\xi_1) = a_1 \xi_1^2 + b_1 \xi_1 + c_1$$

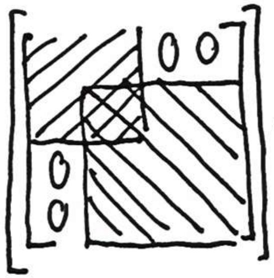
$$a_1 = \frac{\pi (d_{AVE} - d_1)^2}{4 l_e^2}, \quad b_1 = \frac{\pi (d_{AVE} - d_1) d_1}{2 l_e}, \quad c_1 = \frac{\pi d_1^2}{4}$$

$$A_2(\xi_2) = a_2 \cdot \xi_2^2 + b_2 \xi_2 + c_2$$

$$a_2 = \frac{\pi (d_3 - d_{AVE})^2}{4 l_e^2}, \quad b_2 = \frac{\pi (d_3 - d_{AVE}) d_{AVE}}{2 l_e}, \quad c_2 = \frac{\pi d_{AVE}^2}{4}$$

STIFFNESS MARTIX / SOLUTION

$$\underset{5\times5}{[K]} = \begin{bmatrix} [K]_1 & \begin{matrix}0 & 0\\ 0 & 0\end{matrix} \\ \begin{matrix}0 & 0\\ 0 & 0\end{matrix} & [K]_2 \end{bmatrix} \quad , \quad \underset{5\times1}{\{q\}} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{Bmatrix} \quad , \quad \{F\} = \begin{Bmatrix} R_1 \\ 0 \\ 0 \\ 0 \\ F \end{Bmatrix} \quad ; \quad q_1 = 0$$



$$\cdot \begin{Bmatrix} q_2 \\ q_3 \\ q_4 \\ q_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ F \end{Bmatrix} \quad \Rightarrow \quad \begin{Bmatrix} q_2 \\ q_3 \\ q_4 \\ q_5 \end{Bmatrix}$$

DISPLACEMENT :

$$u_1(\xi_1) = \underset{1\times3}{\lfloor N(\xi_1) \rfloor} \cdot \underset{3\times1}{\{q\}_1} \quad , \quad u_2(\xi_2) = \underset{1\times3}{\lfloor N(\xi_2) \rfloor} \cdot \underset{3\times1}{\{q\}_2}$$

ELEMENT SOLUTION

STRAIN :

$$\varepsilon_{x_1}(\xi_1) = \underbrace{\left\lfloor \frac{dN}{d\xi_1} \right\rfloor}_{1\times 3} \cdot \{q\}_1 \quad , \quad \varepsilon_{x_2}(\xi_2) = \underbrace{\left\lfloor \frac{dN}{d\xi_2} \right\rfloor}_{1\times 3} \cdot \{q\}_2$$

STRESS :

$$\sigma_{x_1}(\xi_1) = E \cdot \varepsilon_{x_1}(\xi_1) \quad , \quad \sigma_{x_2}(\xi_2) = E \cdot \varepsilon_{x_2}(\xi_2)$$

INTERNAL FORCES:

$$N_1(\xi_1) = \sigma_{x_1}(\xi_1) \cdot A_1(\xi_1) \quad , \quad N_2(\xi_2) = \sigma_{x_2}(\xi_2) \cdot A_2(\xi_2)$$

ELEMENT SOLUTION

STRAIN :

$$\varepsilon_{x_1}(\xi_1) = \underset{1\times 3}{\left\lfloor \frac{dN}{d\xi_1} \right\rfloor} \cdot \{q\}_1 \quad , \quad \varepsilon_{x_2}(\xi_2) = \underset{1\times 3}{\left\lfloor \frac{dN}{d\xi_2} \right\rfloor} \cdot \{q\}_2$$

STRESS :

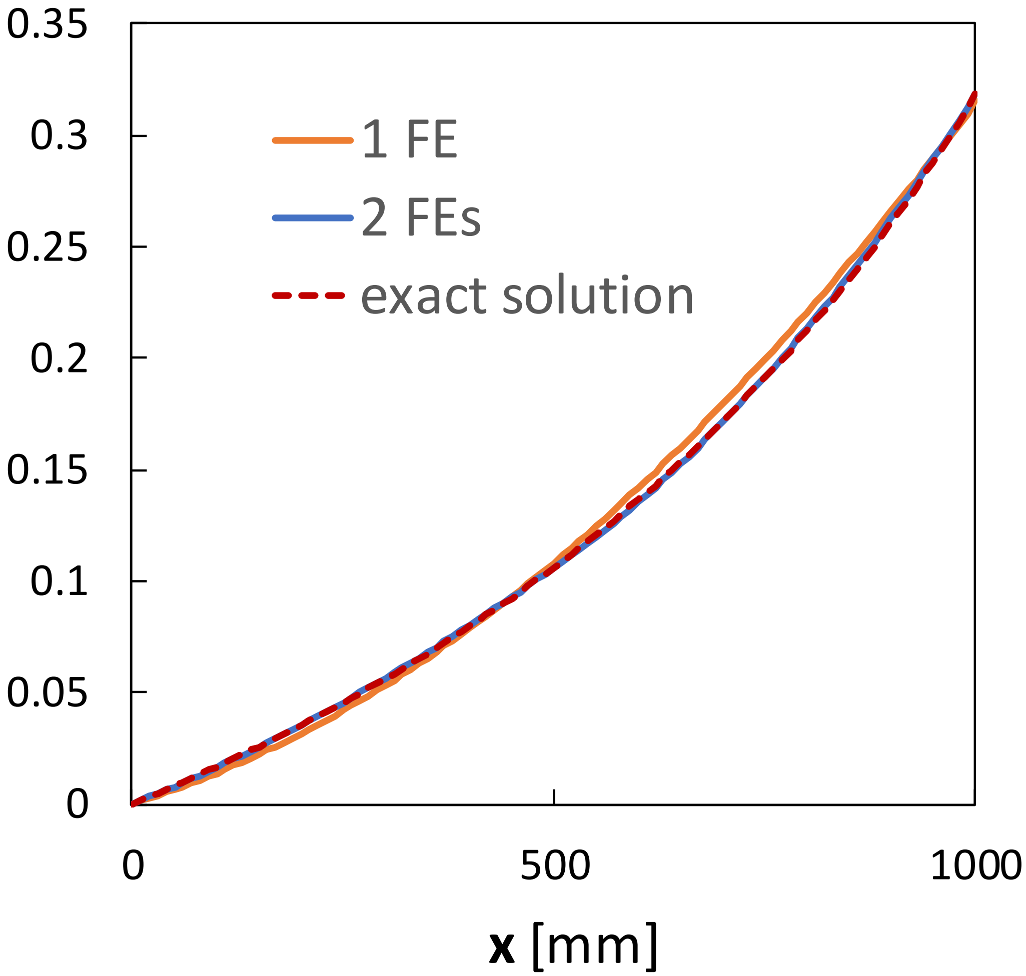
$$\sigma_{x_1}(\xi_1) = E \cdot \varepsilon_{x_1}(\xi_1) \quad , \quad \sigma_{x_2}(\xi_2) = E \cdot \varepsilon_{x_2}(\xi_2)$$

INTERNAL FORCES:

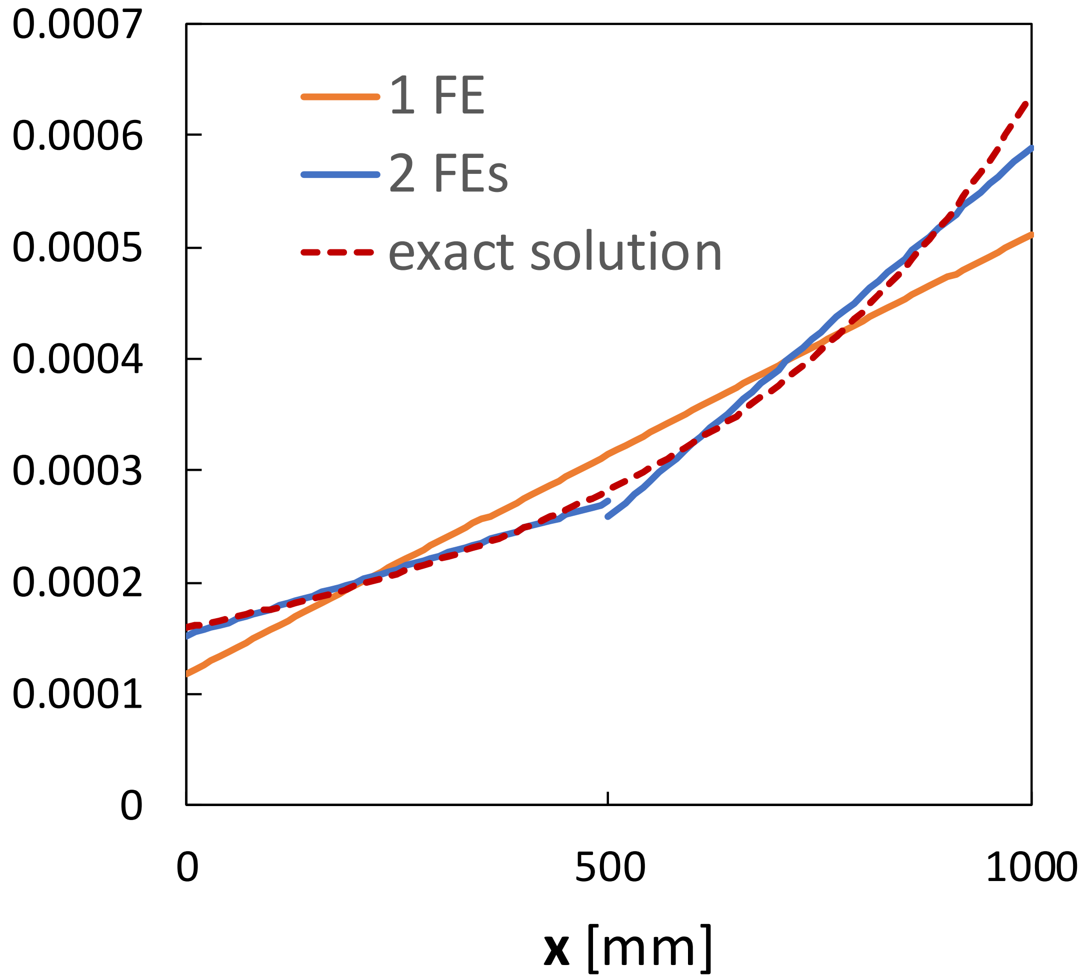
$$N_1(\xi_1) = \sigma_{x_1}(\xi_1) \cdot A_1(\xi_1) \quad , \quad N_2(\xi_2) = \sigma_{x_2}(\xi_2) \cdot A_2(\xi_2)$$

displacement u [mm]

$E = 2 \cdot 10^5 \mathrm{M}Pa$
l = 1000 mm
d_1= 20 mm
d_3= 10 mm
F = 10000 N



strain



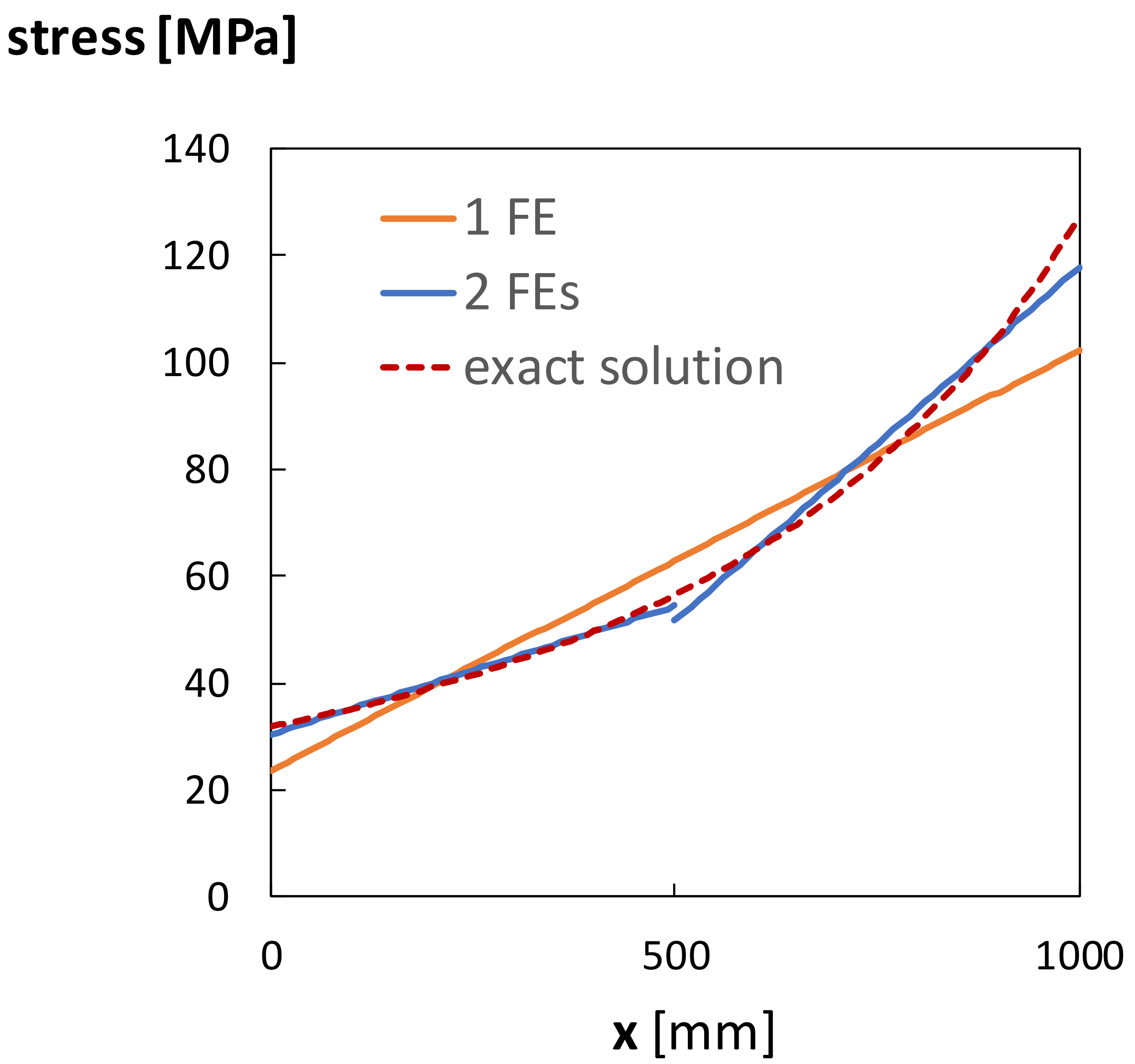
$E = 2 \cdot 10^5 \mathrm{M}Pa$

l = 1000 mm

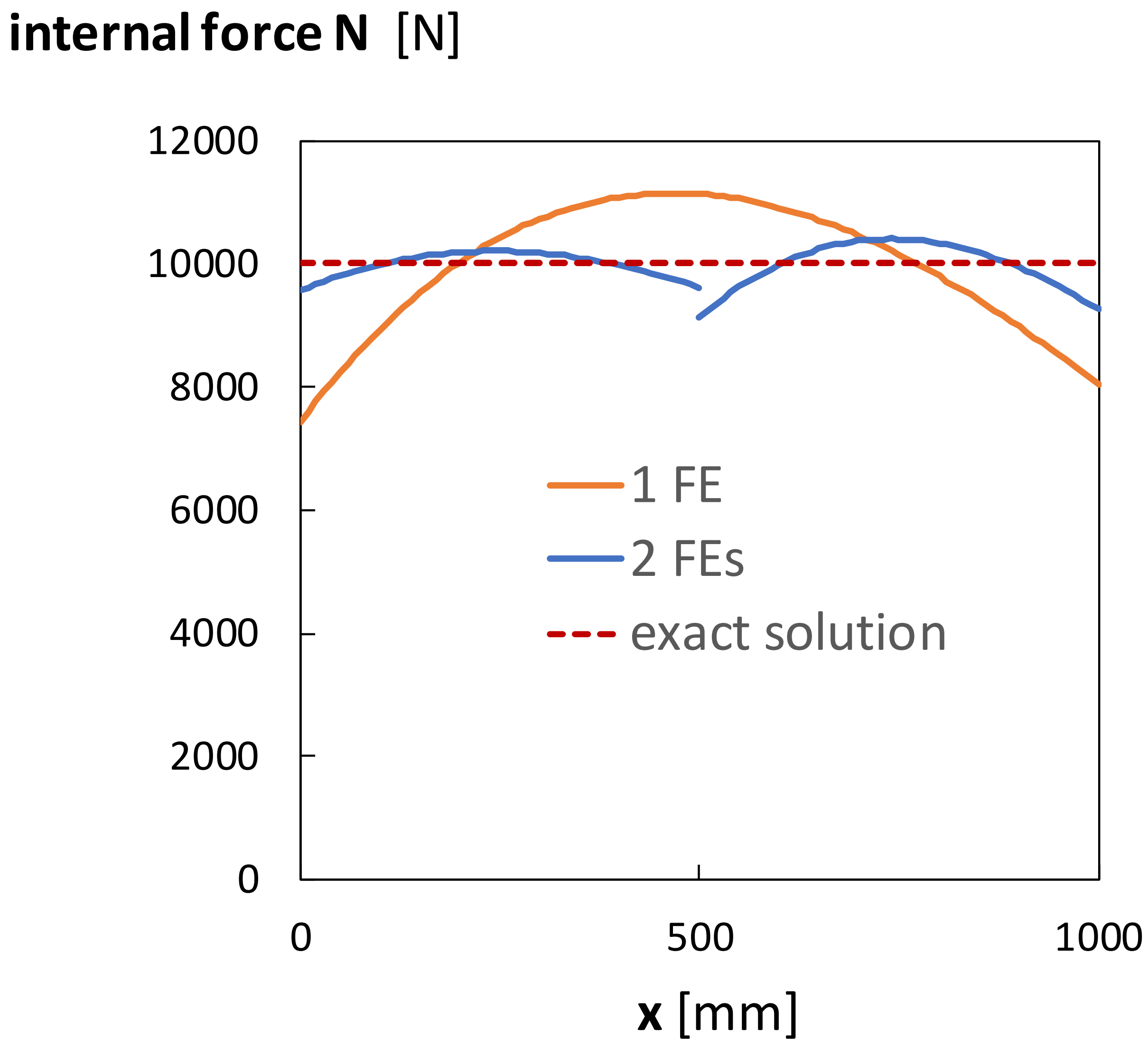
d_1= 20 mm

d_3= 10 mm

F = 10000 N



$E = 2 \cdot 10^5 \mathrm{M}Pa$
l = 1000 mm
d_1= 20 mm
d_3= 10 mm
F = 10000 N



$E = 2 \cdot 10^5 \mathrm{M}Pa$

l = 1000 mm

d_1= 20 mm

d_3= 10 mm

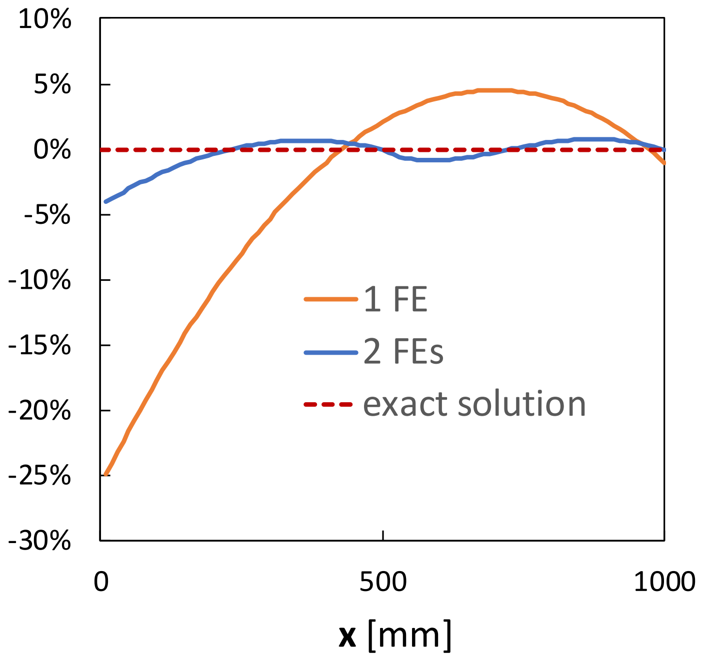
F = 10000 N

reaction

$R_1 = F$

RELATIVE ERRORS

relative error of displacement u



relative error of strain, stress, and internal force